MINIMUM-TIME MOTION PLANNING OF CRANES WITH PARAMETRIC UNCERTAINTY USING LINEAR PROGRAMMING

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Abstract: The problem of minimum-time anti-swing motion planning of cranes with uncertainty in the hoisting speed of the load cable is addressed. The crane is modeled as a trolley-pendulum system for which the trolley acceleration is the control variable. A previously existing method that uses linear programming to solve the problem in the absence of uncertainties has been extended in order to take the uncertainty in the hoisting speed into account. Constraints that impose that the sensitivity of both the load angle and load angular speed with respect to the uncertain parameter to be zero at the final point of the path are included in the linear programming formulation.

Keywords: crane motion planning, minimum-time control, anti-swing control, parametric uncertainties, linear programming

1. INTRODUCTION

The problem of anti-swing motion planning of crane systems has been largely studied in the literature for a long time. A variety of approaches has been proposed to deal with the problem depending on the mathematical model adopted, on the boundary conditions to be satisfied and on the performance index to be minimized (Al-Garni *et al.*, 1995), (Auernig and Troger, 1987), (Cruz and Leonardi, 2012) (Golafshani and Aplevich, 1995), (Lee, 2005), (Liang and Koh, 1997), (Mita and Kanai, 1979), (Sakawa and Shindo, 1982), (Scardua *et al.*, 2003).

It is well known that the search for solutions of constrained optimal control problems is generally a hard task that involves the use of the Pontryagin's Minimum Principle to formulate a nonlinear Two-Point Boundary Value Problem (TPBVP) (Dhanda and Franklin, 2010), (Kamien and Schwartz, 1981), (Seiersted and Sydsaeter, 1977). This is particularly true when the problem contains inequality constraints. Motivated by this fact (Cruz and Leonardi, 2012) proposed a method to solve the minimum-time anti-swing control problem of cranes by using Linear Programming (LP). The goal there was to present a motion planning scheme with no concern to robustness issues. The objective of this paper is to extend such method in order to take into account uncertainties in the load hoisting [2] history.

A lot of work has been carried out on time-optimal problems for flexible structures described by linear time-invariant (LTI) models with uncertainties in natural frequencies and damping coefficients - an exception is (Liu and Singh, 1997), where a nonlinear problem is also dealt with. The approach taken to deal with these uncertainties generally consists of including a constraint in the problem in order to impose that some sensitivity to the uncertain parameters is null. Such sensitivity can be represented by the derivative of a structural mode, as in (Liu and Wie, 1992) and (Wie *et al.*, 1993); or by the residual vibration, as in (Pao and Singhose, 1995), (Singer and Seering, 1990)

[1] Partially supported by CNPq (grant no. 303202/2009-2) and Fapesp (grant no. 2010/11113-2).

[2] All occurrences of *hoisting* in this text can be replaced by *lowering*.

and (Singhose *et al.*, 1996). In particular, in (Liu and Singh, 1997) the sensitivity of final states with respect to the parameters is set to zero.

In practice the hoisting trajectories are usually generated independently of the trolley motions (Lee, 2004). In view of this, as in (Cruz and Leonardi, 2012), the length of the load cable will be considered here as a given function of time. It will be assumed that the cable will be hoisted (lowered) at maximum speed between its initial and final lengths, after which the length will remain constant at the later value.

As a consequence of the assumption that the length of the cable is a given function of time, the crane dynamics can be described by a linear time-varying (LTV) model. It must be emphasized that this is not the case in all the previously cited papers dealing with flexible structures, where all the models are LTI. Hence this is a point that distinguishes this work from them.

(Lee, 2004) studies the motion planning problem for cranes arguing that the dynamical model, which does not depend on the load mass, has no parametric uncertainties. In the present work a more realistic approach is taken by considering that the time-history of the cable length can be affected by uncertainty. In particular, in view of the form adopted to describe the cable hoisting function, it will be assumed that the hoisting speed is the uncertain parameter.

As in (Cruz and Leonardi, 2012), the trolley acceleration is chosen as the control variable. The crane dynamics are initially modeled as a system of differential equations which are then rewritten in discrete-time form assuming that the control variable is a piecewise constant function of the time. To take into account the physical limitations associated to the electromechanical driving system, inequality constraints on both the trolley speed and acceleration are included in the problem. Both the initial and final points of the load path are assumed given where both the trolley and the load must be at rest.

As said before, the objective of this paper is to extend the method proposed by (Cruz and Leonardi, 2012). Following the same line of (Liu and Singh, 1997), this will be done by adding constraints to the problem such that both the load angle and the angular speed of the load cable at the final point are insensitive to uncertainties on the cable hoisting speed.

All variables will be expressed in dimensionless form following the reference (Auernig and Troger, 1987).

2. THE MATHEMATICAL MODEL

The dynamical model adopted in this paper has been taken from (Auernig and Troger, 1987), where the trolley acceleration is the control variable.

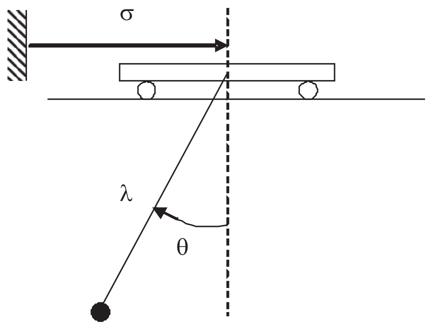

Fig. 1. Sketch of the trolley-pendulum system

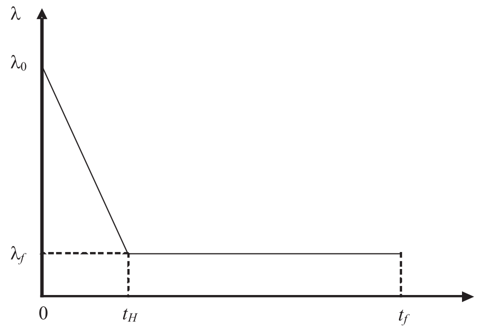

Fig. 2. Cable length as a function of time

The following hypotheses are made: the crane is infinitely rigid, dissipative effects are neglected, the load can be considered a point mass, both the load and the trolley motions occur in the same plane, the cable is massless and inextensible and the influence of hoisting acceleration on load swinging is neglected. Such are usual hypotheses that have been made by many authors (Al-Garni *et al.*, 1995), (Auernig and Troger, 1987), (Benhidjeb and Gissinger, 1995), (Golafshani and Aplevich, 1995), (Liang and Koh, 1997), (Sakawa and Shindo, 1982), (Strip, 1989).

A sketch of the trolley-pendulum is shown in figure 1. As it suggests, σ is the trolley position with respect to a fixed reference, λ is the current length of the load cable and θ is the load cable angle with respect to the vertical direction.

The cable length as a function of time is assumed to have the form illustrated in figure 2. The values of λ_0, λ_f and t_H are given such that

$$\frac{\lambda_f - \lambda_0}{t_H} = v_H, \tag{1}$$

where v_H is the given hoisting speed. t_f represents the final time to be minimized.

Assuming that θ is small, the system dynamical equations can be written as:

$$\ddot{\sigma} = u \tag{2}$$

$$\lambda\ddot{\theta} + 2\dot{\lambda}\dot{\theta} + \theta = u, \tag{3}$$

where over-dots represent time derivatives and u is the trolley acceleration. In fact these LTV equations

are a kinematical model of the trolley-pendulum system which is thus independent of the load mass. See (Auernig and Troger, 1987) for details.

From equations (2)-(3) it is clear that the only source of uncertainty in the model is associated to the load cable length and hoisting speed. Here it will be assumed that the most relevant uncertainty is located in the value of v_H which is the hoisting speed in the interval $(0,\ t_H)$.

3. THE MINIMUM-TIME PROBLEM WITH NO UNCERTAINTY

With no loss of generality, it is assumed that

$$\sigma(0) = 0. \tag{4}$$

Additionally, both the trolley and the load are assumed initially at rest:

$$\dot{\sigma}(0) = 0, \tag{5}$$

$$\theta(0) = 0, \tag{6}$$

$$\dot{\theta}(0) = 0. \tag{7}$$

At the final time t_f the desired position σ_f of the trolley is given and both the trolley and the load must be at rest. Hence,

$$\sigma(t_f) = \sigma_f, \tag{8}$$

$$\dot{\sigma}(t_f) = 0, \tag{9}$$

$$\theta(t_f) = 0, \tag{10}$$

$$\dot{\theta}(t_f) = 0. \tag{11}$$

Limitations of the trolley driving system require that

$$\mid u(t) \mid \leq 1, \tag{12}$$

$$\mid \dot{\sigma}(t) \mid \leq \dot{\sigma}_{max}, \tag{13}$$

for all $t \in [0,\ t_f]$, where $\dot{\sigma}_{max}$ is the given maximum trolley speed. Recall that the unit value in the right-hand side of inequality (12) is a consequence of the fact that u is dimensionless (see (Auernig and Troger, 1987)).

The objective is to minimize t_f with respect to u, i.e.,

$$\min_u t_f. \tag{14}$$

As posed above this is one version of the classical minimum-time motion planning problem of cranes with no uncertainties.

Let the interval $[0,\ t_f)$ be partitioned in n sub-intervals of same length $[t_{i-1},\ t_i)$, $(1 \leq i \leq n)$. If the value of n is chosen sufficiently large, then a stepwise constant function can be taken to approximate the trolley acceleration $u(t)$. Let $u(i)$ denote the value of $u(t)$ in the i-th time interval. (Cruz and Leonardi, 2012)) proposed a method to compute the solution of this discrete-time version of the minimum-time problem given by equations (2)-(14) by solving a sequence of LP problems [3]. For each one of these problems the value of t_f is fixed and the final position of the trolley $\sigma(t_f)$ is maximized subject to the constraints given by equations (2)-(7) and (9)-(13). The value of t_f is iteratively adjusted until $\sigma(t_f)$ is close to σ_f within a given accuracy. For more details and for the proof of convergence of such scheme see (Cruz and Leonardi, 2012).

4. THE MINIMUM-TIME PROBLEM WITH PARAMETRIC UNCERTAINTY

Exactly in the same way as done by (Liu and Singh, 1997) and (Wie *et al.*, 1993), the approach taken here will be to impose that the derivatives of the pendulum angle and angular speed with respect to the uncertain parameter v_H are null at the final time t_f. This will be done in this section by including two equality constraints in the problem posed in the previous section.

Defining

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}, \tag{15}$$

let the state-space model corresponding to equation (3) be written in the usual form [4]

$$\dot{x} = Ax + Bu, \tag{16}$$

where

$$A = A(\lambda, \dot{\lambda}) = \begin{bmatrix} 0 & 1 \\ -1/\lambda & -2\dot{\lambda}/\lambda \end{bmatrix} \tag{17}$$

and

$$B = B(\lambda) = \begin{bmatrix} 0 \\ 1/\lambda \end{bmatrix}. \tag{18}$$

In this first-order form, the model contains the second-order time derivative of θ as $\dot{x}_2$.

Considering increments Δx, $\Delta\lambda$ and $\Delta\dot{\lambda}$ in the functions x, λ and $\dot{\lambda}$, respectively, the following first-order approximation is obtained:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{19}$$

where

$$\mathbf{x} = \begin{bmatrix} x \\ \Delta x \end{bmatrix}, \tag{20}$$

[3] This approximate solution can be viewed as a sub-optimal one where the minimization is performed with respect to the subspace of stepwise constant functions.

[4] It is obvious that equation 2 does not need to be included in the state-space model because σ can be obtained as a function of u immediately by direct integration.

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}, \tag{21}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}, \tag{22}$$

$$\mathbf{A}_{11} = \mathbf{A}_{22} = A, \tag{23}$$

$$\mathbf{A}_{12} = 0_{2\times 2}, \tag{24}$$

$$\mathbf{A}_{21} = \frac{\partial A}{\partial \lambda}\Delta\lambda + \frac{\partial A}{\partial \dot{\lambda}}\Delta\dot{\lambda}, \tag{25}$$

$$\mathbf{B}_1 = B, \tag{26}$$

$$\mathbf{B}_2 = \frac{\partial B}{\partial \lambda}\Delta\lambda. \tag{27}$$

Notice that from equations (19)-(27) it follows that

$$\Delta\dot{x} = A\Delta x + (\frac{\partial A}{\partial \lambda}x + \frac{\partial B}{\partial \lambda}u)\Delta\lambda + (\frac{\partial A}{\partial \dot{\lambda}}x)\Delta\dot{\lambda}. \tag{28}$$

Taking into account that $\Delta x(0) = 0$ and considering $\Delta\dot{\lambda}$ in the form

$$\Delta\dot{\lambda}(t) = \Delta v_H h(t), \tag{29}$$

where Δv_H is the uncertainty in v_H and

$$h(t) = \begin{cases} 1 & (0 \leq t \leq t_H) \\ 0 & (t_H < t \leq t_f) \end{cases}, \tag{30}$$

then it turns out that $\Delta x(t_f) \propto \Delta v_H$. Hence, with no loss of generality, one can take $\Delta v_H = 1$ to compute $\partial x(t_f)/\partial v_H$ as

$$\frac{\partial x(t_f)}{\partial v_H} = \frac{\Delta x(t_f)}{\Delta v_H}. \tag{31}$$

The requirement that $x(t_f)$ be insensitive to uncertainties in v_H can thus be expressed by

$$\Delta x(t_f) = 0. \tag{32}$$

To conclude this section, notice that constraints (10)-(11) together with (32) can be expressed by

$$\mathbf{x}(t_f) = 0, \tag{33}$$

whose left-hand side clearly depends linearly on u. The algorithm proposed by (Cruz and Leonardi, 2012) can thus be applied to the present problem, which differs from the one of the previous section by the addition of the two equality constraints of equation (32). Hence the proof of convergence of the above mentioned sequence of LP problems is exactly the same of (Cruz and Leonardi, 2012).

5. NUMERICAL RESULTS

To illustrate the application of the proposed method, the same set of parameters of the Sepetiba Port, Brazil, cranes reported in (Cruz and Leonardi, 2012) were also used here. See Table 1, where over-hats indicate variables expressed in engineering units.

Parameter	Value
$\hat{\dot{\sigma}}_{max}$	2.4 m/s
$\hat{\lambda}_0$	30 m
$\hat{\lambda}_f$	16 m
$\hat{u}_{max}$	1.0 m/s^2
$\hat{v}_H$	2 m/s
$\hat{\sigma}_f$	46.1 m
n	100

Table 1. Numerical values of the parameters for the example

The minimum value obtained for $\hat{t}_f$ was 24.8 s, which is just 0.5 s larger than the one obtained in (Cruz and Leonardi, 2012) in the absence of the constraints associated to parameter uncertainty.

In figure 3 the trolley acceleration is shown. A typical bang-0-bang pattern is observed. This fact suggests the occurrence of singular intervals. It should be recalled that the study of singular intervals in minimum-time LTI systems is a common topic in many classical references on optimal control (see, for instance, (Kirk, 1970)). Nevertheless, in this paper the system is LTV and not only the control magnitude is upper bounded as usual but also the trolley speed (see eq. 13).

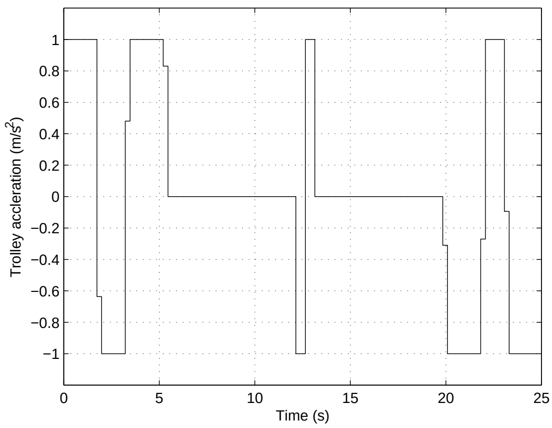

Fig. 3. Trolley acceleration versus time

The trolley speed is shown in figure 4. It can be seen that it is at rest both at the beginning and at the end of the motion, as required.

As expected, both constraints $|\ \hat{u}\ | \leq 1.0$ m/s^2 and $|\ \hat{\dot{\sigma}}\ | \leq 2.4$ m/s have been satisfied.

Figure 5 contains the position of the trolley. Its final value is 46.1 m as desired.

Figures 6 and 7 show, respectively, the angle and the angular speed of the load cable. As required, in both

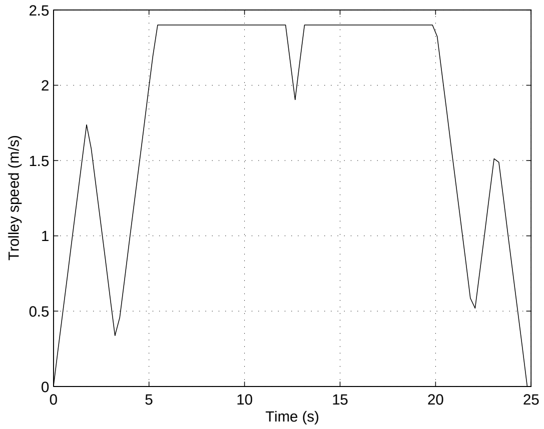

Fig. 4. Trolley speed versus time

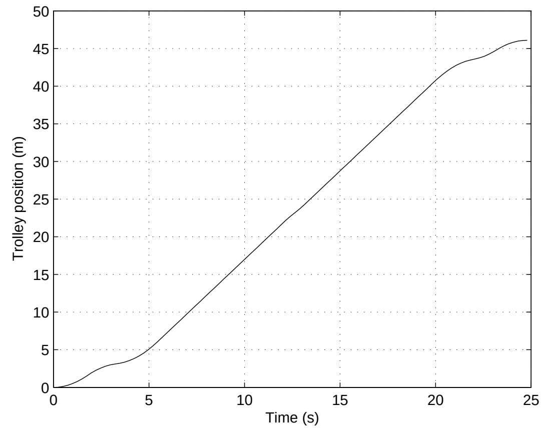

Fig. 5. Trolley position versus time

cases the pendulum is at rest at the vertical position both at the beginning and at the end of the motion.

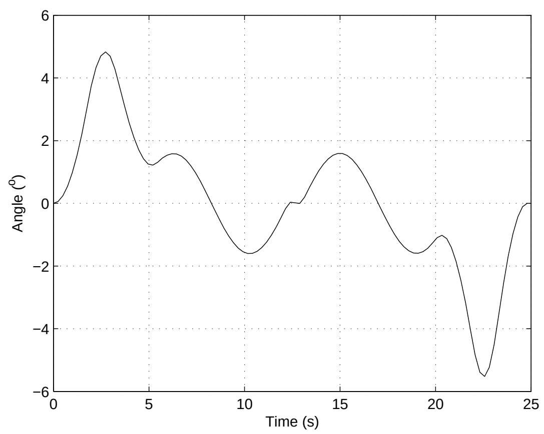

Fig. 6. Load cable angle versus time

To test the sensitivity of the approach proposed here against the one based on the model with no uncertainties, a simulation was run with an error of 5% in the value of the hoisting speed. Figures 8 and 9 show, respectively, the angle and the angular speed of the load cable obtained for the method proposed in this

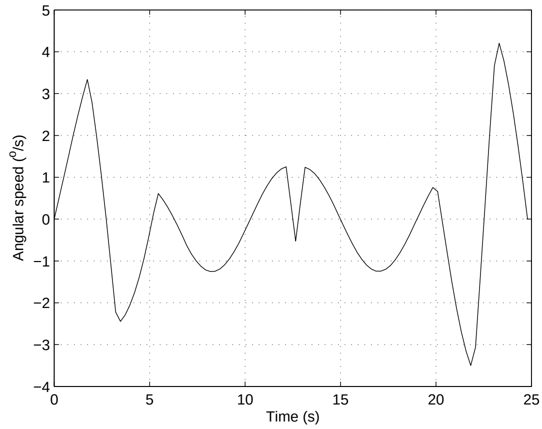

Fig. 7. Load cable angular speed versus time

paper (continuous line) and for the method of (Cruz and Leonardi, 2012) (dashed line). The reduction of about one order of magnitude of the amplitudes of the residual oscillations after 24.8 s shows clearly the improvement produced by the present method in this example.

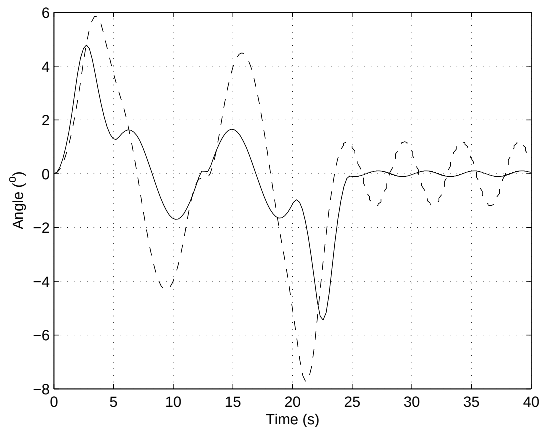

Fig. 8. Load cable angle versus time

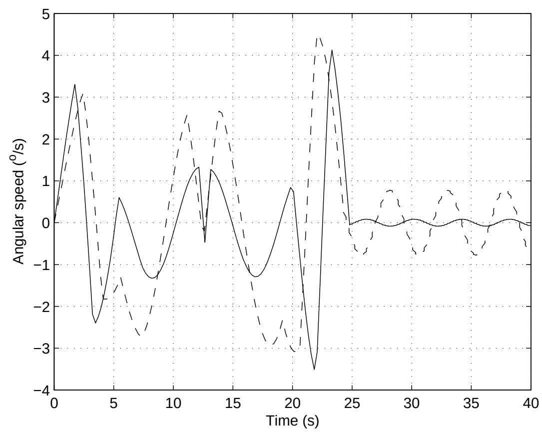

Fig. 9. Load cable angular speed versus time

6. CONCLUSIONS

The LP-based method of reference (Cruz and Leonardi, 2012), that solves the constrained minimum-time anti-swing motion planning of cranes, has been extended to take into account uncertainties in the hoisting speed of the load cable. All the advantages of the LP-based method have been preserved. Among them, the following ones should be mentioned: i) LP algorithms always find the global optimal solution whenever it exists; ii) it is quite simple to deal with equality and inequality linear constraints on both state and control variables. It must be recalled that in general these are *not* the cases of TPBVP algorithms which, in addition, usually require that a "good" initial approximation of the solution be given in some neighborhood of it.

Using a LTV model, the formulation of the uncertain motion planning problem under a LP framework together with the development presented in section 4 are the main points of this paper.

It is clear that the method proposed here could be easily expanded to deal with parametric uncertainties in more complex forms - defined by means of a larger number of parameters - of the function that describes the cable hoisting time history.

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