TRACKING IN SCALE QUAD-ROTORS THROUGH ADAPTIVE AUGMENTATION

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Abstract: This paper illustrates the application of an adaptive flight control architecture to a scale quad-rotor. For autonomous VTOL (Vertical Takeoff and Landing) flight, it is common to separate the control problem into an inner fast loop that controls attitude and an outer slow loop that controls the trajectory of the VTOL. In this paper we augment a conventional PD controller conceived mainly for hovering, with an adaptive element using a real-time tuning single hidden layer neural network in a inner-outer loop combined architecture to account for model inversion error cancelation, issued in the feedback linearization process. The results shown in simulations reveal the superior performance of the augmented controller in tracking maneuvers. Copyright CONTROLO2012.

Keywords: Quad-Rotor, Neural Network, Adaptive Control

1. INTRODUCTION

The potential for unmanned aerial vehicles (UAVs) in applications such as environmental monitoring or fire prevention has been well established. A quad-rotor is an under-actuated, dynamic system with four input forces and six output DOF. Its actuators are four fixed pitch angle rotors. This configuration increases payload capacity and maneuverability. The basic motions of a quad-rotor are generated by varying the rotor speeds of all four motors, thus changing the lift and drag forces. The quad-rotor tilts toward the direction of the slow spinning rotor, which enables acceleration along that direction. The spinning directions of the rotors are set to balance the moments, therefore eliminating the need for a tail rotor. Quad-rotors, as any other UAV's, are affected by aerodynamic forces in strong non-linear coupling, which can be considered uncertain, and also by external disturbances such as wind gusts. In order to conveniently control the quad-rotor it is required to meet the essential stability, robustness and desired dynamic performance, being able to adapt to changing parameters and environmental unmodeled disturbances. Other works present a direct approximate-adaptive control, using CMAC nonlinear approximators, (C. Nicol, 2011). In this paper, a tracking controller is designed for the nonlinear quad-rotor model. In a first stage, the controller consists of two linear proportional plus derivative (PDs) controllers in an inner-outer loop configuration, assuring an ideal tracking capability without external perturbations. The resulting closed loop system is highly sensitive to perturbations, so the initial linear controller is augmented by an adaptive action, introduced by a single hidden layer (SHL) feed forward neural network (NN) acting also in an inner-outer additive arrangement, regarding the linear control. The performance of the augmented system is greatly improved, being capable of adapting to external unmodeled perturbations or even to internal unmodeled dynamics (S. Salazar-Cruz, 2005). The structure of this paper is as follows: section 2 presents some basic ideas on approximate feedback linearization (Kim, 2003; N. Kim, 2007). Section 3 presents the quad-rotor modeling according to a Lagrangian formalism (Pedro Castillo, 2007). Tracking formulation is presented in section 4. In section 5 a case study with simulation results is presented and finally, in section 6 the conclusions are presented.

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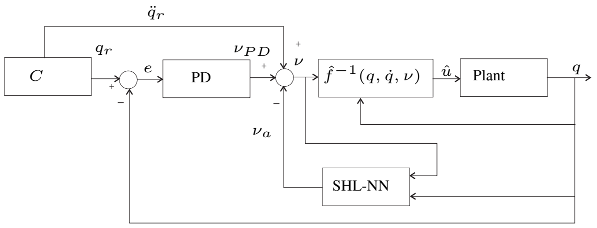

Fig. 1. NN augmented adaptive control architecture

2. APPROXIMATE SYSTEM LINEARIZATION

One common method for controlling nonlinear dynamical systems is based on approximate feedback linearization (Isidori, 1995), which depends on the relative degree of each controlled variable. For newtonian systems like the quad-rotor in a simplified approach, the regulated variables of interest, here represented as the vector q, have relative degree two. The control variables are represented by the vector u.

$$\ddot{q} = f(q, \dot{q}, u) \tag{1}$$

A pseudo control $\nu = f(q, \dot{q}, u)$ is defined such that the dynamic relation with the system state is linear $\ddot{q} = \nu$. Since the function $f(q, \dot{q}, u)$ is not exactly known, an approximation $\nu = \hat{f}(q, \dot{q}, u)$ is used which is invertible regarding u resulting in

$$\ddot{q} = \nu + \Delta(q, \dot{q}, u) \tag{2}$$

where the modeling error is represented by $\Delta(q, \dot{q}, u)$ $= f(q, \dot{q}, u) - \hat{f}(q, \dot{q}, u)$, so the effective actuation can be calculated as

$$\hat{u} = \hat{f}^{-1}(q, \dot{q}, \nu) \tag{3}$$

Supposing in (2) that $\Delta(q, \dot{q}, u) = 0$ we can proceed in the stabilization problem, choosing a linear controller, a PD for instance, that will locally solve the regulation problem. A SHL neural network with conveniently adapted weights will be responsible for modeling error cancelation. Including a command path generator C, the former linear controller can be augmented through the architecture depicted in Fig. 1. The pseudo control signal in (2) is the sum of three components

$$\nu = \ddot{q}_r + \nu_{PD} - \nu_a \tag{4}$$

where $\ddot{q}_r$ is generated by C, ν_{PD} is generated by the PD controller and ν_a is generated by the adaptive element introduced to compensate for the model inversion error. The tracking error is computed as

$$e = \begin{bmatrix} q_r - q \\ \dot{q}_r - \dot{q} \end{bmatrix} \tag{5}$$

and the PD controller can be represented by $\nu_{PD} =$ $[K_p \ K_d]e$, so the tracking error dynamics is given by $\dot{e} = Ae + B(\nu_a - \Delta)$ with

$$A = \begin{bmatrix} O & I \\ -K_p & -K_d \end{bmatrix}, \quad B = \begin{bmatrix} O \\ I \end{bmatrix} \tag{6}$$

where I and O are a suitable identity and null matrices respectively.

2.1 *Adaptive Element*

The adaptive element is implemented by a SHL-NN with conveniently tuned weights V, W such that

$$\nu_a = W^\top \bar{\sigma}(V^\top \bar{q}) \tag{7}$$

with $\bar{q} = [\nu, q]$. Given a sufficient number of hidden layer neurons and appropriate inputs, it should be possible to train a SHL-NN (K. Hornik, 1989) on line to cancel the effect of Δ. The weight matrices are

$$\begin{aligned} V &= \begin{pmatrix} v_{0,1} & v_{0,2} & \cdots & v_{0,n_2} \\ v_{1,1} & v_{1,2} & \cdots & v_{1,n_2} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n_1,1} & v_{n_1,2} & \cdots & v_{n_1,n_2} \end{pmatrix} \\ W &= \begin{pmatrix} w_{0,1} & w_{0,2} & \cdots & w_{0,n_3} \\ w_{1,1} & w_{1,2} & \cdots & w_{1,n_3} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n_2,1} & w_{n_2,2} & \cdots & w_{n_2,n_3} \end{pmatrix} \end{aligned} \tag{8}$$

Here n_1, n_2, n_3 are the number of inputs, hidden layer nodes and outputs. Also $\bar{\sigma}(\xi) = (1, \sigma(\xi_1), \cdots, \sigma(\xi_{n_1}))^\top$. The scalar function σ is a sigmoidal activation function and α is chosen according to (K. Hornik, 1989)

$$\sigma(\xi) = \frac{1}{1 + e^{-\alpha\xi}} \tag{9}$$

2.2 *Contractibility*

The transformation (7) must be contractive regarding ν_a. Note that Δ depends on ν_a through ν, whereas ν_a has to be designed to cancel Δ. Hence the existence and uniqueness of a fixed point solution for $\nu_a =$ $\Delta(q, \dot{q}, \nu_a)$ must be assumed. A sufficient condition is to ascertain that the map $\nu_a \to \Delta(q, \dot{q}, \nu_a)$ is a contraction over the entire input domain of interest, or $\|\partial\Delta/\partial\nu_a\| < 1$. This condition is equivalent to (Kim, 2003).

$$0 < \frac{1}{2}\left|\frac{\partial f}{\partial u}\right| < \left|\frac{\partial \hat{f}}{\partial u}\right| < \infty \tag{10}$$

Consider the system (1), the inverse law (3) and the contractibility property, as well as the adaptation laws

$$\begin{aligned} \dot{W} &= -\left((\bar{\sigma} - \bar{\sigma}' V^\top \bar{q}) r^\top + \kappa \|e\| W\right) \Gamma_W \\ \dot{V} &= -\Gamma_V \left(\bar{q}(r^\top W^\top \bar{\sigma}') + \kappa \|e\| V\right) \end{aligned} \tag{11}$$

where

$$\bar{\sigma}'(\hat{z}) \equiv \frac{\partial \bar{\sigma}(z)}{\partial z}\Big|_{z=\hat{z}} \tag{12}$$

is the Jacobian matrix and $r = e^\top PB$. Also $P \succ 0$ is the unique positive definite solution for the Lyapunov equation $A^\top P + PA + Q = 0$ for any convenient $Q \succ 0$. A and B are defined in (6). Given (11) with $\Gamma_W \succ 0$, $\Gamma_V \succ 0$ and $\kappa > 0$, according to (Nardi, 2000; Shin, 2005) uniform boundedness of the tracking error e is assured. Consider now an aerial vehicle modeled as a nonlinear system, through the following set of equations

$$\begin{aligned} \dot{p} &= v \\ \dot{v} &= T(p, v, q, \omega, u) \\ \dot{q} &= Q(q, \omega) \\ \dot{\omega} &= R(p, v, q, \omega, u) \end{aligned} \tag{13}$$

where, $p \in R^3$ is the position vector, $v \in R^3$ is the velocity of the vehicle, $q \in R^4$ is the attitude quaternion and $\omega \in R^3$ is the angular velocity and $u \in R^4$ is the control action. T represents the translational dynamics and R represents the attitude dynamics. Q represents the quaternion propagation equations. The state vector x can now be defined as $x = (p^\top, v^\top, q^\top, \omega^\top)^\top$. Approximate feedback linearization of the system represented by (13) is achieved by introducing the following transformation:

$$\begin{aligned} \dot{v}_{des} &= \hat{T}(p, v, q_{des}, \omega, u_v) \\ \dot{\omega}_{des} &= \hat{R}(p, v, q, \omega, u_\omega) \end{aligned} \tag{14}$$

where, $\dot{v}_{des}$, $\dot{\omega}_{des}$ are commonly referred to as the pseudo-control and represent desired accelerations. Here, $\hat{T}$ and $\hat{R}$ represent an available approximation of $T(\cdot)$ and $R(\cdot)$. Additionally, $\{u_v,\ u_\omega\}$ and q_{des} are the control inputs and attitude that are predicted to achieve the desired pseudo-control. When $\hat{T}(\cdot)$ and $\hat{R}(\cdot)$ are chosen such that they are invertible, the desired control and attitude may be written as

$$\begin{aligned} u_v &= \hat{T}_u^{-1}(p, v, \omega, \dot{v}_{des}) \\ q_{des} &= \hat{T}_q^{-1}(p, v, \omega, \dot{v}_{des}) \\ u_\omega &= \hat{R}^{-1}(p, v, \omega, \dot{\omega}_{des}) \end{aligned} \tag{15}$$

here $\hat{T}(\cdot) = \hat{T}_u(\cdot) + \hat{T}_q(\cdot)$. Adopting the values of $\{u_v,\ u_\omega\}$, q_{des} from the the inverse control law results in the following closed-loop translational and attitude dynamics

$$\begin{aligned} \dot{v} &= \dot{v}_{des} + \Delta_v(x, u) \\ \dot{\omega} &= \dot{\omega}_{des} + \Delta_\omega(x, u) \end{aligned} \tag{16}$$

3. TRACKING ERROR DYNAMICS

Defining e as the tracking error

$$e = \begin{bmatrix} p_r - p \\ v_r - v \\ q_r \ominus q \\ \omega_r - \omega \end{bmatrix} \quad \text{and} \quad \dot{e} = \begin{bmatrix} \dot{p}_r - \dot{p} \\ \dot{v}_r - \dot{v} \\ \omega_r - \omega \\ \dot{\omega}_r - \dot{\omega} \end{bmatrix} \tag{17}$$

where, $\ominus : R^4 \times R^4 \to R^3$, is a function such that given two quaternions, issues an error angle vector with three components (Johnson, 2000). Let us define also the linear control laws plus adaptive corrections.

$$\begin{aligned} \dot{v}_{des} &= K_d^v(v_r - v) + K_p^v(p_r - p) - \nu_v \\ \dot{\omega}_{des} &= K_d^\omega(\omega_r - \omega) + K_p^\omega(q_r \oplus q_{des}) \ominus q - \nu_\omega \end{aligned} \tag{18}$$

Here, $\oplus : R^4 \times R^4 \to R^4$ represents the rotation addition (Johnson, 2000), obtained as the product of two quaternions, and $\{\nu_v,\ \nu_\omega\}$ are control actions with the purpose of $\{\Delta_v(x, u),\ \Delta_\omega(x, u)\}$ cancelation. Now, developing (16), (17) and (18) the error dynamics results on

$$\dot{e} = Ae + B(\nu - \Delta(x, u)) \tag{19}$$

with

$$A = \begin{bmatrix} 0_3 & I_3 & 0_3 & 0_3 \\ -K_p^v & -K_d^v & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 & I_3 \\ 0_3 & 0_3 & -K_p^\omega & -K_d^\omega \end{bmatrix},\ B = \begin{bmatrix} 0_3 & 0_3 \\ I_3 & 0_3 \\ 0_3 & 0_3 \\ 0_3 & I_3 \end{bmatrix} \tag{20}$$

where $\{K_d^v, K_p^v, K_d^\omega, K_p^\omega\} > 0$, $I_3, 0_3$ are respectively identity and null rank 3 matrixes. The dynamics of (19) should be stable by choosing $\{K_d^v, K_p^v, K_d^\omega, K_p^\omega\} > 0$ such that A remains Hurwitz. Associated with the tracking error dynamics given in (19), is the Lyapunov function $A^\top P + PA + Q = 0$. Choosing positive definite

$$Q^\bullet = \begin{bmatrix} K_d^\bullet (K_p^\bullet)^2 & 0_3 \\ 0_3 & K_d^\bullet K_p^\bullet \end{bmatrix} \tag{21}$$

with $\bullet = \{v, \omega\}$ the Lyapunov equation solution gives

$$P = \begin{bmatrix} P^v & 0_6 \\ 0_6 & P^\omega \end{bmatrix} \tag{22}$$

$$P^\bullet = \begin{bmatrix} (K_p^\bullet)^2 + \frac{1}{2} K_p^\bullet (K_d^\bullet)^2 & \frac{1}{2} K_p^\bullet K_d^\bullet \\ \frac{1}{2} K_p^\bullet K_d^\bullet & K_p^\bullet \end{bmatrix} \tag{23}$$

4. REFERENCE MODEL

A reasonable choice for the reference model dynamics is given by

$$\begin{aligned} \dot{v}_r &= K_p^v(p_c - p_r) + K_d^v(v_c - v_r) \\ \dot{\omega}_r &= K_p^\omega(q_c \oplus q_{des}) \ominus q_r + K_d^\omega(\omega_c - \omega_r) \end{aligned} \tag{24}$$

where $\{p_c,\ v_c,\ q_c,\ \omega_c\}$ are the trajectory commanded values.

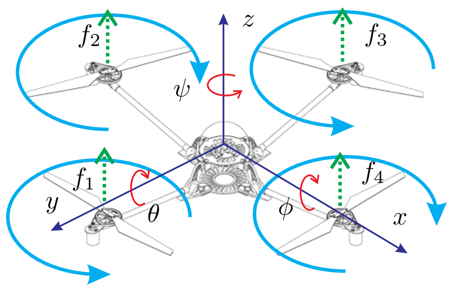

Fig. 2. Quadrotor representation

5. SIMPLIFIED MODELING - APPLICATION TO THE SCALE QUAD-ROTOR

Regarding the quad-rotor as a rigid body, the equations of motion can be derived by applying Newton-Euler formalism.

$$\begin{aligned} \frac{d}{dt}\left(m\rho(q)v^b\right) &= \rho(q)f^b \\ \frac{d}{dt}\left(\rho(q)\mathcal{J}\,\omega^b\right) &= \rho(q)\tau^b \end{aligned} \tag{25}$$

with $\mathcal{J}$ the inertia matrix, v^b the velocity in body coordinates, $\rho(q)$ the rotation matrix, ω^b the angular velocity in body coordinates, and τ^b the external applied torques in body coordinates. After some derivations we obtain

$$\begin{aligned} \dot{p} &= v^p \\ \dot{v}^p &= \frac{1}{m}\rho(q)f_1^b + \frac{1}{m}f_0 \\ \dot{q} &= q^{\triangleright}\omega^b \\ \dot{\omega}^b &= \mathcal{J}^{-1}(\tau^b - \omega^b \wedge \mathcal{J}\,\omega^b) \end{aligned} \tag{26}$$

here v^p is the velocity in inertial coordinates, $f^b = f_1^b + \rho(q)^\top f_0$ and

$$f_1^b = \begin{bmatrix} 0 \\ 0 \\ u \end{bmatrix},\quad f_0 = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \tag{27}$$

and $\tau_\eta = (\tau_\phi, \tau_\theta, \tau_\psi)$ being the moments regarding the local reference frame. Here $q = \{q_0, q_1, q_2, q_3\}$ represents the rotation attitude quaternion, with $q_0 = \cos(\gamma/2)$, $\{q_1, q_2, q_3\} = \overrightarrow{q}\sin(\gamma/2)$. The angle γ establishes the rotation performed about the axis given by $\overrightarrow{q}$.

$$\rho(q) = (q_0^2 - \langle\overrightarrow{q}, \overrightarrow{q}\rangle)I_3 + 2q^{\square} - 2q_0 q^{\times} \tag{28}$$

is the rotation matrix obtained using Rodriguez formula (Xuxi Zhang, 2010), and

$$\begin{aligned} \langle\overrightarrow{q}, \overrightarrow{q}\rangle &= \sum_{i}^{3} q_i^2 \\ q^{\square} &= \begin{bmatrix} q_1^2 & q_1q_2 & q_1q_3 \\ q_1q_2 & q_2^2 & q_2q_3 \\ q_1q_3 & q_1q_2 & q_3^2 \end{bmatrix} \\ q^{\times} &= \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \\ q^{\triangleright} &= \frac{1}{2}\begin{bmatrix} -q_2 & -q_3 & -q_4 \\ q_1 & -q_4 & q_3 \\ q_4 & q_1 & -q_2 \\ -q_3 & q_2 & q_1 \end{bmatrix} \end{aligned} \tag{29}$$

The approximate operators $\hat{T}(\cdot)$, $\hat{R}(\cdot)$ are obtained as follows. Supposing for $f_1^b = (f_{1_x}^b, f_{1_y}^b, f_{1_z}^b)^\top$ that $\left|f_{1_z}^b\right| \gg \max\{\left|f_{1_x}^b\right|, \left|f_{1_y}^b\right|\}$, follows that $\dot{v}_{des} \cong \frac{1}{m}\rho(q)(0,\ 0,\ f_{1_z}^b)^\top + \frac{1}{m}f_0$, attaining

$$\begin{aligned} \dot{v}_{des} - \frac{1}{m}f_0 = \frac{u}{m}&\begin{bmatrix} 2(q_1q_3 - q_0q_2) \\ 2(q_0q_1 + q_2q_3) \\ q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \\ &= \begin{bmatrix} \dot{v}_{des_x} \\ \dot{v}_{des_y} \\ \dot{v}_{des_z} + g \end{bmatrix} \end{aligned} \tag{30}$$

comparing (28) with the rotation Euler-123 matrix in which $(\phi,\ \theta,\ \psi)$ are associated with (roll, pitch, yaw) (see (31))

$$\begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ c\psi s\theta s\phi - c\phi s\psi & c\phi c\psi + s\theta s\phi s\psi & c\theta s\phi \\ c\phi c\psi s\theta + s\phi s\psi & c\phi s\theta s\psi - c\psi s\phi & c\theta c\phi \end{bmatrix} \tag{31}$$

equations (30) are equivalent to

$$\frac{u}{m}\begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} = \begin{bmatrix} \dot{v}_{des_x} \\ \dot{v}_{des_y} \\ \dot{v}_{des_z} + g \end{bmatrix} \tag{32}$$

After some trigonometric considerations and according to (15) we obtain

$$\begin{aligned} |u_v| &= m\,|\dot{v}_{des}| \\ \phi_{des} &= \arctan(\dot{v}_{des_y}/(\dot{v}_{des_z} + g)) \\ \theta_{des} &= \arcsin(\dot{v}_{des_x}/\,|\dot{v}_{des}|) \\ \psi_{des} &= 0 \end{aligned} \tag{33}$$

with $(\phi_{des},\ \theta_{des},\ \psi_{des})$ it is straightforward the obtention of q_{des}. Following with $\hat{R}(\cdot)$ and considering $\dot{\omega}_{des} = \mathcal{J}^{-1}(\tau^b - \omega^b \wedge \mathcal{J}\,\omega^b)$ with $\left|\omega^b\right| \ll 1$ follows that $\dot{\omega}_{des} \cong \mathcal{J}^{-1}\tau^b$. The τ^b moments can be modeled in a first degree of approximation, without considering rotor dynamics (see Fig. 2), as:

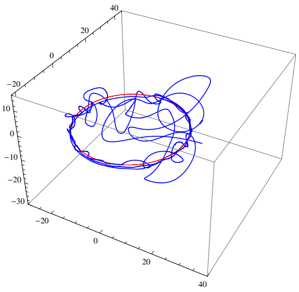

Fig. 3. Tracking with PD control without adaptation. Payload perturbation

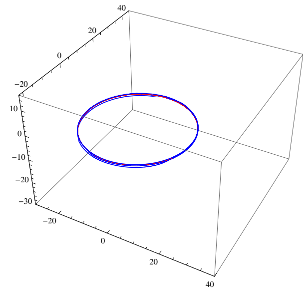

Fig. 4. Tracking PD control with adaptation. Payload Perturbation

$$
\begin{aligned}
u_v &= \sum_{i=1}^{4} f_i \\
f_i &= b_o u_i^2 \\
u_\omega &= (\tau_\phi, \tau_\theta, \tau_\psi)^\top \\
\tau_\psi &= (d_o/b_o)(f_2 + f_4 - f_1 - f_3) \\
\tau_\theta &= l(f_4 - f_2) \\
\tau_\phi &= l(f_3 - f_1)
\end{aligned} \tag{34}
$$

where f_i are the lifting forces in each rotor, u_i the corresponding angular velocities, l the diagonal distance between axes of the respective rotors, and d_o, b_o are *drag* and *thrust* factors, respectively. Once defined u_v, the relationship between τ^b and $\{u_i\}$ is straightforward, where $C_o = 1/(4B_oD_ol)$.

$$
\begin{bmatrix} u_1^2 \\ u_2^2 \\ u_3^2 \\ u_4^2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2lb_o} & 0 & \frac{1}{2lb_o} & 0 \\ 0 & -\frac{1}{2lb_o} & 0 & \frac{1}{2lb_o} \\ -\frac{1}{4d_o} & \frac{1}{4d_o} & -\frac{1}{4d_o} & \frac{1}{4d_o} \\ \frac{1}{4b_o} & \frac{1}{4b_o} & \frac{1}{4b_o} & \frac{1}{4b_o} \end{bmatrix} \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \\ u_v \end{bmatrix} \tag{35}
$$

6. SIMULATION RESULTS

Adopting a reference path given by $P_r = \{\rho_r \cos(\Omega_r t), \rho_r \sin(\Omega_r t), 0, 0, 0, \Omega_r t\}$ and parameters $\rho_r = 20$, $\Omega_r = 0.1, m = 2, l = 0.4, \mathcal{J}_{11} = 0.5, \mathcal{J}_{22} = 0.3, \mathcal{J}_{33} = 0.2, n_1 = 9, n_2 = 3, n_3 = 3, \Gamma_{V_\xi} = 20I, \Gamma_{W_\xi} = 20I, \Gamma_{V_\eta} = 10I, \Gamma_{W_\eta} = 10I, K_p^v = 4, K_d^v = 2I, K_p^\omega = 18I, K_d^\omega = 2I, \kappa_\xi = 2, \kappa_\eta = 4, \alpha = 1, V$ is 17×3 and W is 4×3. The validity of the proposed controller can be noticed in Figs. 3 to 6, where the tracking maneouvers are performed with payload and intertial variations according to Figs. 7, 9. Figs. 8, 10 show typical evolutions of the neural network weigths. All length units are in m.

7. OBTAINING THE ADAPTATION LAWS

Let us consider the Lyapunov function

$$
\mathcal{V}(e, \tilde{V}, \tilde{W}) = \frac{1}{2}\left(e^\top P e + \mathrm{tr}\left(\tilde{W}^\top \Gamma_W^{-1} \tilde{W}\right) + \mathrm{tr}\left(\tilde{V}^\top \Gamma_V^{-1} \tilde{V}\right)\right) \tag{36}
$$

In order to obtain the adaptation equations (11) we must follow the steps required to proof that, on the

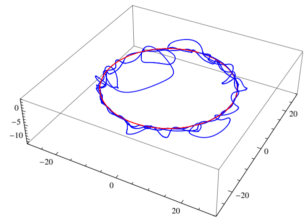

Fig. 5. Tracking with PD control without adaptation. Inertia Perturbation

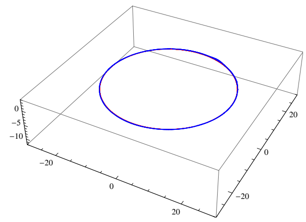

Fig. 6. Tracking PD control with adaptation. Inertia Perturbation

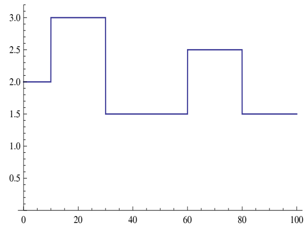

Fig. 7. Payload $m(t)$ perturbation history

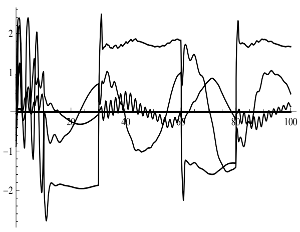

Fig. 8. W_ξ weights evolution under payload $m(t)$ perturbations

error orbits, the condition $\dot{\mathcal{V}} \leq 0$ is satisfied, as explained in (Suresh K. Kannan, 2002). The following steps are given in order to show the parameters regarding an adequate tuning of the controller. The details of the proof of convergence follow the above mentioned reference. Let us consider $\epsilon = \nu_a^* - \Delta =$

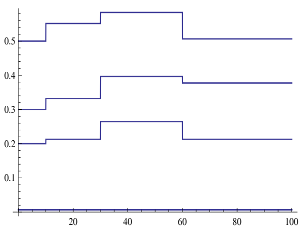

Fig. 9. Payload $\mathcal{J}(t)$ perturbation history

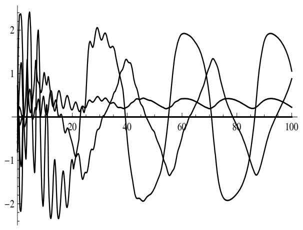

Fig. 10. W_ξ weights evolution under inertia $\mathcal{J}(t)$ perturbations

$W^{*\top}\bar{\sigma}(V^{*\top}\bar{q}) - \Delta$, where W^*, V^* are the optimum values that best approximate Δ. The error dynamics is

$$\begin{aligned} \dot{e} \;=\; & Ae + B\left(W^{*\top}\bar{\sigma}(V^{\top}\bar{q})\right. \\ & \left.-W^{\top}\bar{\sigma}(V^{*\top}\bar{q}) + \epsilon\right) \end{aligned} \quad (37)$$

Defining now $\tilde{W} = W - W^*$, $\tilde{V} = V - V^*$ and using the Taylor series expansion of σ with respect to V in the neighborhood of V^*, which is the optimum value, we obtain

$$\begin{aligned} \dot{e} \;=\; & Ae + B\Big(\tilde{W}^{\top}(\sigma - \sigma' V^{\top}\bar{q}) \\ & +W^{\top}\sigma'\tilde{V}^{\top}\bar{q} + w\Big) \end{aligned} \quad (38)$$

with $w = \epsilon - W^{*\top}\left(\sigma^* - \sigma + \sigma'\tilde{V}^{\top}\bar{q}\right) + \tilde{W}^{\top}\sigma' V^{*\top}\bar{q}$. Substituting now (11) and (38) in the expression of $\dot{V}$ we have

$$\begin{aligned} \dot{V} \;=\; & -\frac{1}{2}e^{\top}Qe + e^{\top}PBw \\ & -\kappa\,\|e\|\,\mathrm{tr}\left(\tilde{Z}^{\top}Z\right) \end{aligned} \quad (39)$$

where

$$Z = \begin{pmatrix} V & 0 \\ 0 & W \end{pmatrix}, \quad \tilde{Z} = Z - Z^* \quad (40)$$

Using $\mathrm{tr}(\tilde{Z}^{\top}Z) \leq \left\|\tilde{Z}\right\|\|Z^*\| - \left\|\tilde{Z}\right\|^2$ and following (Suresh K. Kannan, 2002) there exist $a_0, a_1, c_3, \kappa > \|PB\|\,c_3$ such that

$$\begin{aligned} \dot{V} \;=\; & -\frac{1}{2}\lambda_{min}(Q)\,\|e\|^2 \\ & -(\kappa - \|PB\|\,c_3)\,\|e\|\left\|\tilde{Z}\right\|^2 \\ & +a_0\,\|e\| + a_1\,\|e\|\left\|\tilde{Z}\right\| \end{aligned} \quad (41)$$

and, with $Z_m = \frac{a_1 + \sqrt{a_1^2 + 4a_0(\kappa - \|PB\|c_3)}}{\kappa - \|PB\|c_3}$,

$$\|e\| \geq \frac{a_0 + a_1 Z_m}{\frac{1}{2}\lambda_{min}(Q)} \Rightarrow \dot{V} \leq 0 \quad (42)$$

Thus for convenient initial conditions, the tracking error e is ultimately uniformly bounded.

8. CONCLUSIONS

This paper presents the adaptive augmentation of a linear tracking controller. This augmentation prevents and cancels unmodeled perturbations, making possible the adoption of a simplified plant model. This is specially worth in UAVs and particularly in quadrotors. The simulations confirm the robustness of this methodology.

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