# A COMPARATIVE ANALYSIS OF TRACKING QUALITY FOR CDM AND PID CONTROL

**Wojciech Giernacki**

*Institute of Control and Information Engineering, Poznan University of Technology,*
*3a Piotrowo St., 60-965 Poznan, Poland*
*Wojciech.Giernacki@put.poznan.pl*

Abstract: A tracking quality of CDM and PID control in systems with SMPO (stable, minimum-phase, oscillatory) plant are presented and compared in this paper. Constraints of control signal amplitude and step type of disturbance were considered in simulation tests, where for the synthesis of PID controller *Matlab 7.0/Simulink Response Optimization* libraries were used. The synthesis of robust control system was conducted with the use of coefficient diagram method (CDM) algorithm. These two methods were compared on a benchmark of trajectory tracking. Comments of the results obtained for these methods are given. In all of simulation tests, a higher quality of tracking was obtained in CDM system. 

Keywords: Coefficient diagram, Coefficient diagram method, Pole placement, Tracking quality, Robust control.

## 1. INTRODUCTION

From the early twenties of the twentieth century to the present day, PID control is constantly the most common type of control in industry because of the simplicity of tuning, and satisfactory results in many practical realizations (Åström and Murray, 2008; Bhattacharyya *et al*., 2005; Lee I. *et al*., 2009). It should be noted that PID control in spite of the many advantages, in general does not provide optimal control neither the stability of the system, what can be considered as the greatest disadvantage.

A basic conception of the coefficient diagram method (CDM) was developed by (Manabe, 1998). General ideas on possible applications of CDM algorithm are discussed, e.g., in (Bigdeli and Haeri, 2008; Budiyono and Sudiyanto, 2007; Kongratana *et al*., 2007). This method based on an idea of using relationship between the obtained closed-loop system time characteristics and the placement of poles of its characteristic polynomial on the complex plane *s*. In CDM it is assumed that each control is a compromise between the desired dynamics of the system and its stability. The stability is a fundamental aim of all control methods, but varied systems with different types of plants require a different character of control and in general, these requirements may be opposed to one another and introduce constraints. Moreover, the influence of external disturbances on the system is also important and must be reduced by controller.

In this paper, the efficiency of PID control and proposed coefficient diagram method were compared. CDM in the considered variant, provides the stability, robustness and dynamics of the designed system in the presence of disturbances and control signal constraints. For both methods, the need of the tracking efficiency compare is reasonable, because these concepts connects the engineering simplicity of the controller synthesis with their scientifically and mathematically proven ground.

The paper is structured as follows. In Section 2, the controller synthesis is presented, as well as coefficient diagram (CD) - a tool for assessment of the system stability, robustness and dynamics. Numerical, comparative simulations of a tracking quality for CDM and PID control in systems with SMPO plant are presented in Section 3. In Section 4, a conclusion of the results is given.

## 2. CDM PROCEDURE

Mathematical ground of CDM are in detail discussed e.g., in (Manabe, 1998), therefore here only the CDM algorithm is presented. The control system under consideration is illustrated in Fig. 1, where *F(s)* – input numerator polynomial of the controller transfer function, *A(s)/B(s)* – numerator/denominator polynomial of the controller transfer function, *N(s)/D(s)* – numerator/denominator of the plant transfer function, *r(t)* – reference signal, *e(t)* – control error signal, *v(t)/u(t)* – unconstraint/constraint control signal, *z(t)* – external disturbance signal, *y(t)* – output signal, *m(t)* – measured output noise signal.

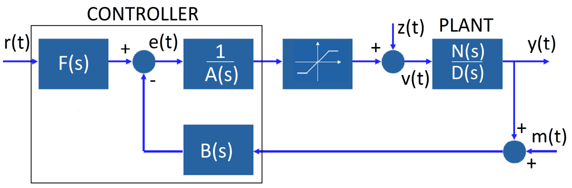


Fig. 1. Block diagram of a control system.

The synthesis of robust controller by the CDM algorithm from Fig. 2, uses procedure and CD presented below, see also (Giernacki, 2010).

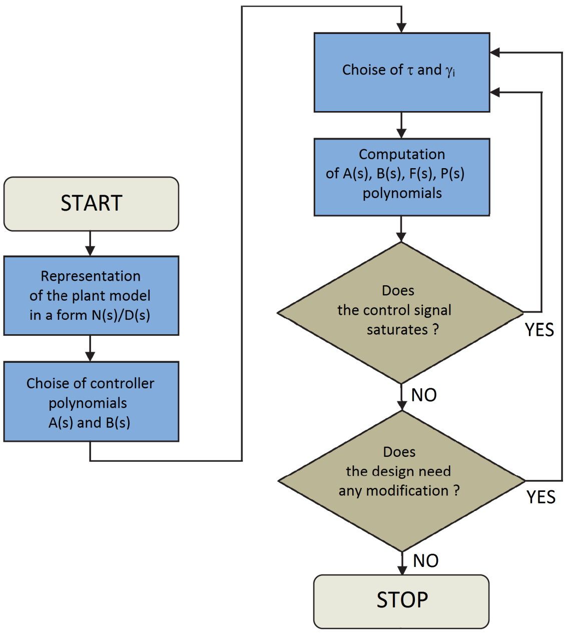


Fig. 2. CDM algorithm.

### *2.1 Controller synthesis.*

- Notation of plant model with the use of transfer function (1):

$$\frac{N(s)}{D(s)} = \frac{n_l s^l + n_{l-1} s^{l-1} + \ldots + n_1 s + n_0}{d_m s^m + d_{m-1} s^{m-1} + \ldots + d_1 s + d_0}, \qquad (1)$$

where: *l* – degree of *N(s)* polynomial (less or equal to *m* – degree of *D(s)* polynomial).
For the plant models with delay to obtain the model transfer function (1) Padé approximation is used.

- Choice of controller structure

Based on the analysis of expected disturbances, degrees of polynomials *A(s)* and *B(s)* are chosen according to Table 1.

Table 1 The choice of transfer function polynomials degrees due to expected type of disturbances

| Type of disturbance | Degree of *A(s)* | Degree of *B(s)* | Degree of *P(s)* | Condition |
|---|---|---|---|---|
| None | *m-1* | *m-1* | *2m-1* | - |
| Step | *m* | *m* | *2m* | $l_0=0$ |
| Ramp | *m+1* | *m+1* | *2m+1* | $l_0= l_1=0$ |
| Impulse/ sinusoidal | *m-1* | *m-1* | *2m-1* | - |

If the expected type of disturbance may not be specified, or may be many types of disturbances, then it is recommended to assume controller of a higher order and gradually reduce the degrees of controller model polynomials in the following steps of the algorithm (Fig. 2). Controller polynomials, respectively degree: *p* and *q* must be written in the forms (2)-(3):

$$A(s) = \sum_{i=0}^{p} l_i s^i , \qquad (2)$$

$$B(s) = \sum_{i=0}^{q} k_i s^i . \qquad (3)$$

- Choice of τ and $\gamma_i$ values

The CDM uses the relationship (4) between the equivalent of time constant ($\tau$) – used to build the characteristic polynomial ($P_T$) and the expected time of step response ($t_s$):

$$\tau = t_s / (2.5 \sim 3) . \qquad (4)$$

An advantage of the proposed algorithm is Manabe standard form (5), which is a vector specifying the stability indices ($\gamma_i$). This vector represents the stability of the system on a CD and defines the polynomial $P_T(s)$, which should be used to ensure the requirements of the system dynamics in the first iteration of the algorithm. Standard forms should be treated as initial setting values of each index of stability, which may be tuned in the next iteration – detail in (Manabe, 1998).

$$\underline{\gamma_i} = [2.5 \quad 2 \quad 2 \quad \ldots \quad 2]^T, \qquad (5)$$

for $i=1,\ldots,n-1$ and $\gamma_0=\gamma_n=\infty$, where $n$ – degree of characteristic polynomial.
To specify numerically and graphically (on CD) the stability limit of the system, equation (6) for limits of stability is used:

$$\gamma_i^* = \frac{1}{\gamma_{i-1}} + \frac{1}{\gamma_{i+1}}, \qquad (6)$$

for $i=2,\ldots,n-2$, and:

$$\gamma_1^* = \frac{1}{\gamma_2}, \quad \gamma_{n-1}^* = \frac{1}{\gamma_{n-2}} .$$

- Calculation of *P(s)*, *F(s)*, *A(s)* and *B(s)*

Characteristic polynomial of the system is defined by the equation (7):

$$P(s) = D(s)A(s) + N(s)B(s) = \sum_{i=0}^{n} a_i s^i \qquad (7)$$

and polynomial *F(s)* equation (8):

$$F(s)\big|_{s=0} = \frac{P(s)}{N(s)}\bigg|_{s=0}. \qquad (8)$$

The equivalent of time constant and stability indices building the target characteristic polynomial (9), which is compared to equation (7), thus from diophantine equation (10) numerical values of controller coefficients ($l_i$ and $k_i$) may be calculated.

$$P_T(s) = a_0 \left\{ \left[ \sum_{i=2}^{n} \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^{j}} \right) (\tau s)^i \right] + \tau s + 1 \right\} \qquad (9)$$

$$P(s) = P_T(s) \qquad (10)$$

If values $a_0$, $\tau$, $\gamma_i$ are set in advance, the issue in question refers to solving the classic problem of pole placement.

- Recurrence of CDM algorithm

Calculation of controller and characteristic polynomial parameters of closed-loop system (with or without the control signal amplitude constraint) may be realized once or several times. The possible option of procedure recurrence depends only on the fact, whether a satisfactory control quality was obtained (according to previously chosen criterion e.g. size of the overshoot, saturation of control signal, setting time of output signal, specified limit of stability). In a situation where quality control is unsatisfactory, usually by a reduction of stability limit or extension of the expected time of step response, algorithm may be recurred. The CD analysis is useful in this part of procedure.

### *2.2 Coefficient diagram.*

In synthesis and analysis of the control system based on the CDM algorithm, half-logarithmic coefficient diagram is used (Fig. 3), where the vertical axis logarithmically shows the coefficients of the characteristic polynomial ($a_i$), stability indices ($\gamma_i$), stability limits ($\gamma_i$*) and the equivalent time constant ($\tau$), while the horizontal axis shows the $i$ values corresponding to each coefficient.

In Fig. 3 color marks were introduced:

- numerical values ($a_i$) of the coefficients of characteristic polynomial $P_T$ – blue curve,
- numerical values ($k_i$) of the coefficients of *B(s)* polynomial – black,
- numerical values of stability indices ($\gamma_i$) – green,
- numerical values of stability limits ($\gamma_i$*) – red,
- equivalent of time constant ($\tau$) – purple.

By analogy with the Bode and Nyquist plots, coefficient diagram provides the necessary information about the system robustness, stability and dynamics. The degree of convexity, which is obtained from coefficients of the characteristic polynomial, gives a measure of stability, while the general inclination of the curve gives a measure of the speed of response (Manabe, 1998). The variation of the shape of the $a_i$ curve due to plant parameter variation is a measure of robustness (Hamamci and Ucal, 2002).

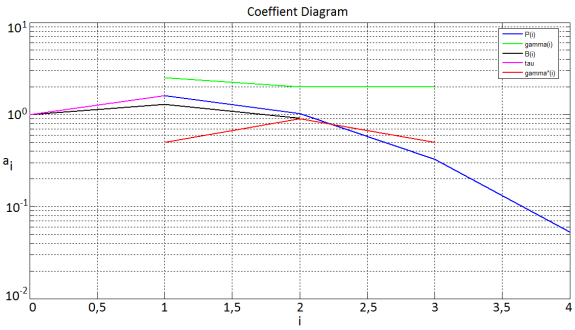


Fig. 3. Coefficient diagram of CDM system with SMPO model.

For larger values of stability indices convexity of the blue curve on the CD - created from the characteristic polynomial based on the values of stability index, increases and the system has a larger stability limit, what may be graphically presented as well. Accordingly, for values $\gamma_i$ and $\gamma_i^*$ vertical distance between the green and red curves is a measure of system stability (if the distance for each $i$ increases, then the system has bigger stability limit). It should be noted that the system is stable only if the curves do not cross each other and the green curve is above the red curve (Manabe, 1998).

Assessment of system robustness is based on the mutual position of blue and black curves. If the black curve is below the blue curve, then the system is more robust to parametric uncertainty – robustness increases when the curves are closer to each other.

Dynamics of the system is characterized by the time constant equivalent. The system is characterized by higher dynamics for smaller values of $\tau$ - in diagram it corresponds to a larger angle of purple curve inclination. Analysis of the time constant equivalent is also important in the case of control signal constraints. If the control signal is saturated, then the key issue in the control system synthesis is to return to the stage of selection of the expected equivalent time constant value to increase this value (to slow down the expected step response) and retry the CDM algorithm.

## 3. SIMULATIONS

The quality of a tracking in CDM and PID systems with SMPO plant model has been compared. Transfer function of model is written in the form (11):

$$G(s) = \frac{N(s)}{M(s)} = \frac{1}{s^2 + 0.2s + 1} \; . \tag{11}$$

For the CDM controller it was expected that the assumed time of step response (without overshoot) will be up to 4 seconds (time constant equivalent from equation (4) is then equal to 1.6 seconds). Selection of the controller polynomials based on the Table 1 and a potential character of step disturbances was anticipated. From equation (5) a vector of stability indices according to the Manabe, standard form was chosen, to provide the system marked by an appropriate robustness and has necessary stability limit. For such choice of CDM algorithm parameters, results of CDM controller tuning are presented in Table 2.

Table 2 Results of CDM controller tuning

| Vector $\gamma_i$ from Manabe form | $[2.5\ 2\ 2]^T$ |
|---|---|
| Vector of values $\gamma_i^*$ | $[0.5\ 0.9\ 0.5]^T$ |
| Polynomial coefficients values vector $P_T(s)$ | $[0.0524\ 0.3277\ 1.0240\ 1.6000\ 1.0000]^T$ |
| Polynomial coefficients values vector $A(s)$ | $[0.0524\ 0.3172\ 0]^T$ |
| Polynomial coefficients values vector $B(s)$ | $[0.9081\ 1.2828\ 1.0000]^T$ |

PID controller was set with the use of *Simulink Response Optimization* library of *Matlab 7.0* environment. Optimization of the controller tuning was conducted for the assumed time of step response (defined similarly as for the CDM) excluding constraints of control signal. PID controller settings were obtained as:

- gain of proportional part $k_P=0.2082$,
- gain of integral part $k_I=0.9084$,
- gain of derivative part $k_D=0.8662$.

Parameters of PID and CDM controllers remained constant in all performed simulations. To tracking quality assessment, except for time courses, integral quality indices (12)-(13) were used:

- IAE - integral of absolute value of error:

$$IAE = \int_0^\infty |e(t)| dt, \tag{12}$$

- ISE - integral of error squared:

$$ISE = \int_0^\infty |e(t)|^2 dt. \tag{13}$$

### 3.1 Tracking of set-point signal in nominal systems.

In systems without disturbances and constraints of control signal amplitude, tracking quality of set rectangular signal (amplitude equal to 1, period to 60 [s], control horizon to 200 [s]), was tested. Obtained signals are shown in Fig. 4.

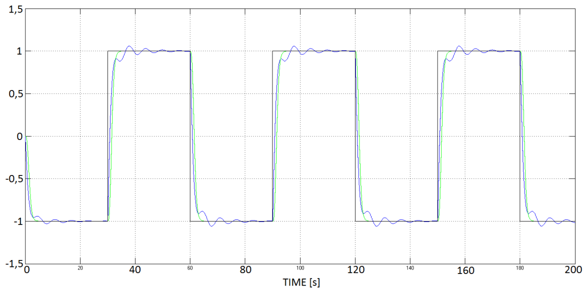


Fig. 4. Tracking of *y(t)* signals (CDM – green color, PID – blue) after *r(t)* signal (black) in nominal systems.

Analysis of signals from Fig. 4 confirms the fulfillment of the design intent for the CDM controller (expected signal without overshoot was obtained), but a tracking of reference signal in the PID system is characterized by damped oscillations of a not high amplitude, what makes evident that it is a system with a plant of oscillatory nature. Table 3 also confirms the better tracking quality in CDM system (lower values of IAE and ISE indices).

Table 3 Integral quality indices for a tracking of set-point signal in nominal CDM and PID systems

| | CDM | PID |
|---|---|---|
| IAE | 4.309 | 17.01 |
| ISE | 0.5757 | 14.59 |

### 3.2 Tracking of set-point signal in disturbed systems.

An additional external disturbance with step change of amplitude level over time has been introduced to systems add the point 3.1. The quality of tracking was verified in the simulation with similar parameters as in Section 3.2. The research results is in Fig. 5 presented – in both cases, the systems follow a set-point signal, but in the case of CDM control, the quality is definitely better – appearing disturbances are damped much more quickly and there are no oscillations of *y(t)* signal. Analysis of the IAE and ISE index values (Table 4) suggests no significant differences in the quality of tracking for CDM system in comparison to the nominal system, but the differences of index values ISE for PID system are significant and well reflect the different dynamics of *e(t)* signal changes, which is in Fig. 6 presented.

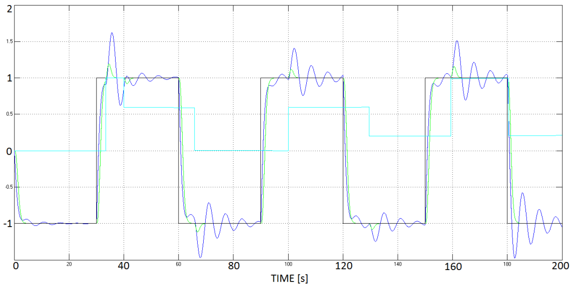


Fig. 5. Tracking of *y(t)* signals (CDM – green color, PID – blue) after *r(t)* signal (black) in disturbed systems (disturbance – cyan).

Table 4 Integral quality indices for a tracking of set-point signal in disturbed CDM and PID systems

| | CDM | PID |
|---|---|---|
| IAE | 4.943 | 28.84 |
| ISE | 0.6003 | 18.11 |

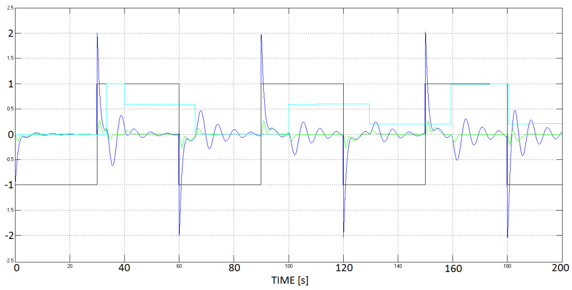


Fig. 6. Time courses of *e(t)* signals (CDM – green color, PID – blue) for tracking of *r(t)* signal (black) in disturbed systems (disturbance – cyan).

### 3.3 *Tracking of set-point signal in disturbed systems at constraints.*

In the next stage of a controllers comparison, an impact of control signal amplitude constraints on tracking quality was considered. The same cut-off levels of control signal were imposed - respectively ±2; ±1.5 and ±1 (although the amplitude of *u(t)* signal in CDM system not exceed the value of 2 – this value was left as a reference in the graphical comparison). Signals from Fig. 7 and Fig. 8 as well as values of IAE and ISE indices from Table 5 were obtained.

Table 5 Integral quality indices for a tracking of set-point signal in disturbed CDM and PID systems with constraints of control signal amplitude ($u_{max}$)

| $u_{max}$ | | CDM | PID |
|---|---|---|---|
| <-2,2> | IAE | 4.943 | 40.49 |
| | ISE | 0.6003 | 29.43 |
| <-1.5,1.5> | IAE | 4.961 | 42.00 |
| | ISE | 0.6014 | 31.14 |
| <-1,1> | IAE | 5.117 | 43.57 |
| | ISE | 0.8199 | 34.14 |

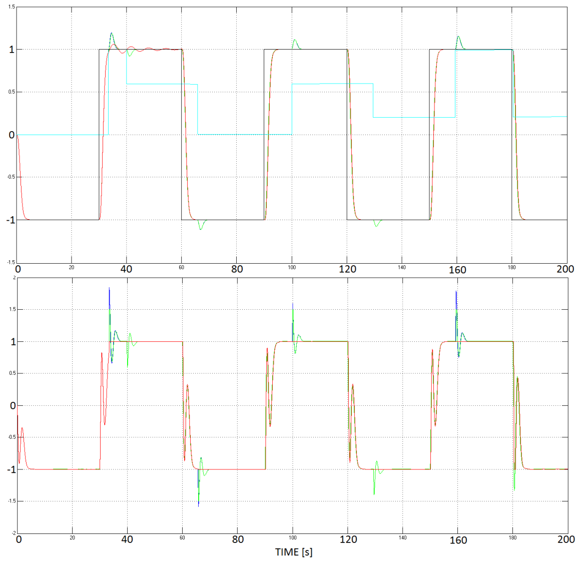


Fig. 7. Signals: *y(t)* (a) and *u(t)* (b) with $u_{max}$=±2 – blue curve, $u_{max}$=±1.5 – green, $u_{max}$=±1 – red in disturbed CDM system (disturbance – cyan) for tracking problem of *r(t)* signal (black).

In system with CDM controller the constraint of control signal reduces the maximum value of *y(t)* signal, but there is a tendency to oscillation of that signal (increasing values of IAE and ISE).

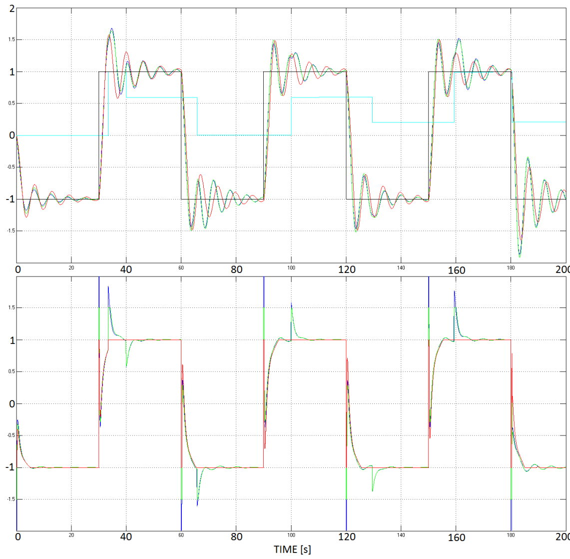


Fig. 8. Signals: *y(t)* (a) and *u(t)* (b) with $u_{max}$=±2 – blue curve, $u_{max}$=±1.5 – green, $u_{max}$=±1 – red in disturbed PID system (disturbance – cyan) for tracking problem of *r(t)* signal (black).

In system with PID controller the constraint of control signal reduces also the maximum value of *y(t)* signal, but increases the time of tracking of the *r(t)* signal (increasing values of IAE and ISE).

## 4. CONCLUSIONS

All of the conducted simulation tests have shown that, in the case of tracking for systems with an oscillatory nature plant, better quality generated the coefficient diagram method. CDM controller dumps the disturbances both faster and more efficiently than PID controller with optimization of its sets with the use of *Simulink Response Optimization*.

## REFERENCES


Åström, K.J. and R.M. Murray (2008). *Feedback Systems*, 408. Princeton University Press, New Jersey.

Bhattacharyya, S.P., Datta, A. and G.J. Silva (2005). *PID Controllers for Time-Delay Systems*, 330. Birkhäuser, Boston.

Bigdeli, N., and M. Haeri (2008). AQM controller design for networks supporting TCP vegas: A control theoretical approach. *ISA Transactions*, **47**, 143-155.

Budiyono, A., and T. Sudiyanto (2007). An Algebraic Approach for the MIMO Control of Small Scale Helicopter. In: *Proc. ICIUS'07*, pp. 64-72, Bali, Indonesia.

Giernacki, W., (2010). *Robust adaptive control with control signal constraint based on CDM algorithm (Ph.D. dissertation in Polish)*, 165. Poznan University of Technology, Poznan.

Hamamci, S.E., and A. Ucar (2002). A robust model-based control for uncertain systems. *Transactions of the Institute of Measurement and Control*, **24**, 331-345.

Lee I., Lee J. and S.W. Sung (2009). *Process Identification and PID Control*, 352. John Wiley & Sons (Asia) Pte Ltd, Singapore.

Kongratana, V., A. Numsomran, P. Roengruen, T. Suksri, and S. Thuengsripan (2007). Smith Predictor Design by CDM for Temperature Control System. In: *Proc. ICCAS'07*, pp. 1472-1477, Seoul, Korea.

Manabe, S., (1998). Coefficient Diagram Method. In: *Proc. IFAC Automatic Control in Aerospace*, pp. 199-210, Seoul.